Green dyadics in uniaxial bianisotropic-ferrite medium by cylindrical vector wavefunctions

1997 J. Phys. A: Math. Gen. 30 573

(http://iopscience.iop.org/0305-4470/30/2/020)

PII: S0305-4470(97)74862-5

Green dyadics in uniaxial bianisotropic-ferrite medium by cylindrical vector wavefunctions

Dajun Cheng†, Wei Ren‡ and Ya-Qiu Jin†

† Wave Scattering and Remote Sensing Centre, Department of Electronic Engineering, Fudan University, Shanghai 200433, People's Republic of China

‡ Department of Applied Mathematics, University of Electronic Science and Technology of China, Chengdu 610054, People's Republic of China

Received 21 May 1996, in final form 4 September 1996

Abstract. The uniaxial bianisotropic-ferrite medium, which can be fabricated by polymer synthesis techniques, is a generalization of the well-studied chiral medium. It has potential applications in the design of antireflection coating, antenna radomes, and novel microwave components. In the present investigation, based on the concept of spectral eigenwaves, eigenfunction expansion of the Green dyadics in this class of materials is formulated in terms of the cylindrical vector wavefunctions. The formulations are greatly simplified by analytically evaluating the integrals with respect to the spectral longitudinal and radial wavenumbers, respectively. The analysis indicates that the solutions of the source-incorporated Maxwell's equations for a homogeneous uniaxial bianisotropic-ferrite medium are composed of two (or four) eigenwaves travelling with different wavenumbers. Each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave. The Green dyadics of planarly and cylindrically multilayered structures consisting of uniaxial bianisotropic-ferrite media can be straightforwardly obtained by applying the method of scattering superposition and appropriate electromagnetic boundary conditions respectively. The resulting formulations, which can be theoretically verified by comparing their special forms with existing results, provide fundamental basis to analyse the physical phenomena of unbounded and multilayered uniaxial bianisotropic-ferrite media.

1. Introduction

The concept of vector wavefunctions was first proposed by Hansen [1] to solve source-free Maxwell's equations in isotropic media. This vector-wavefunction approach has been intensively developed by Felsen and Marcuvitz [2], Morse and Feshbach [3], and Tai [4], to investigate the source-incorporated electromagnetic boundary value phenomena of isotropic media. It has been discovered that for some types of electromagnetic boundary value problems of isotropic media (e.g. microstrip wraparound antennas [5], circular-shaped microwave radiators [6, 7], and excitations of cylindrical waveguides and cavities [8]), field representations and Green dyadics by the cylindrical vector wavefunctions are more useful than those by the planar vector wavefunctions. Recently, field representations by the cylindrical vector wavefunctions were presented for the source-free gyroelectric chiral media [9], composite chiral-ferrite media [10], reciprocal uniaxial bianisotropic media [11], and uniaxial bianisotropic-ferrite media [12]. However, analytic solutions to the source-incorporated Maxwell's equations in any given complex media still need to be developed, so as to provide methodological convenience in studying the physical phenomena of these materials.

0305-4470/97/020573+13$19.50

The Green dyadic in one of the basic tools that are used to solve source-incorporated Maxwell's equations [4, 13, 14]. It is useful both in analysing radiation problems [4, 14, 15] and in constructing integral equations for scattering phenomena [16, 17]. The general representation of the Green dyadic expressed in terms of an expansion of the vector wavefunctions are required to study Raman and fluorescent scattering by active molecules embedded in a particle [18, 19], as well as to establish T-matrix formulation from Huygens's principle and extinction theorem [20, 21]. Furthermore, eigenfunction expansion of the Green dyadics could also provide fundamental insight into the physical process of the material under consideration. However, much effort is still required in order to obtain the Green dyadics in any given complex media when expressed in the full eigenfunction expansion of the vector wavefunctions.

With recent advances in polymer synthesis techniques, increasing attention is being attracted to the analysis of interaction between electromagnetic waves and novel microwave material [11, 22, 23], in order to determine how to use these materials to provide better solutions to current engineering problems. Among these novel microwave materials, one should mention the uniaxial bianisotropic-ferrite medium [12] because of its potential applications in microwave technology, antenna design, and particularly in antireflection coating. In practice, a uniaxial bianisotropic-ferrite medium with linear magnetoelectric interaction can be fabricated by arranging two types of microstructures (short helices and Ω-shaped elements) in the same magnetized ferrite host material. From a phenomenological point of view, a homogeneous uniaxial bianisotropic-ferrite medium can be characterized by the set of constitutive relations [12]. (In the following analysis, the harmonic $\mathrm{e}^{\mathrm{i}\omega t}$ time dependence of the fields and exciting sources is assumed and suppressed throughout.)

$$\boldsymbol{D}(\boldsymbol{r}) = \bar{\varepsilon} \cdot \boldsymbol{E}(\boldsymbol{r}) + \bar{\boldsymbol{\xi}} \cdot \boldsymbol{H}(\boldsymbol{r}) \tag{1a}$$

$$\boldsymbol{B}(\boldsymbol{r}) = \bar{\boldsymbol{\mu}} \cdot \boldsymbol{H}(\boldsymbol{r}) + \bar{\boldsymbol{\zeta}} \cdot \boldsymbol{E}(\boldsymbol{r}) \tag{1b}$$

where

$$\bar{\varepsilon} = \varepsilon_t \bar{\boldsymbol{I}}_t + \varepsilon_z \boldsymbol{e}_z \boldsymbol{e}_z \tag{2a}$$

and

$$\bar{\boldsymbol{\mu}} = \mu_t \bar{\boldsymbol{I}}_t + \mu_z \boldsymbol{e}_z \boldsymbol{e}_z - \mathrm{i} g \boldsymbol{e}_z \times \bar{\boldsymbol{I}}_t \tag{2b}$$

are permittivity and permeability dyadics, respectively.

$$\bar{\boldsymbol{\xi}} = \mathrm{i}(\mu_0 \varepsilon_0)^{1/2} (-\alpha \bar{\boldsymbol{I}}_t + \beta \boldsymbol{e}_z \times \bar{\boldsymbol{I}}_t - \gamma \boldsymbol{e}_z \boldsymbol{e}_z) \tag{2c}$$

and

$$\bar{\boldsymbol{\zeta}} = \mathrm{i}(\mu_0 \varepsilon_0)^{1/2} (\alpha \bar{\boldsymbol{I}}_t + \beta \boldsymbol{e}_z \times \bar{\boldsymbol{I}}_t + \gamma \boldsymbol{e}_z \boldsymbol{e}_z) \tag{2d}$$

are the magnetoelectric pseudo-dyadics. Here, $\bar{\boldsymbol{I}}_t = \boldsymbol{e}_x \boldsymbol{e}_x + \boldsymbol{e}_y \boldsymbol{e}_y$ denotes the transverse unit dyadic, and $\boldsymbol{e}_j$ represents the unit vector in the j direction. Instead of three constitutive parameters for the well-studied chiral media [24, 25], we are facing a medium with eight constitutive parameters. It is apparent that the constitutive dyadics of the medium satisfy the nonreciprocity conditions [26] and uniformity constraint condition [27]. For the lossless uniaxial bianisotropic-ferrite medium, the constitutive parameters ε_t, ε_z, μ_t, μ_z, g, α, β, and γ are all real, which are assumed through the present consideration.

It should be mentioned that the constitutive relations (1) and (2) are formulated in the complex domain, not in the real and physical space. In physical space, the real and measurable fields $\mathcal{D}(\boldsymbol{r}, t)$, $\mathcal{B}(\boldsymbol{r}, t)$, $\mathcal{E}(\boldsymbol{r}, t)$, and $\mathcal{H}(\boldsymbol{r}, t)$ are defined to be the real parts of $\boldsymbol{D}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}$, $\boldsymbol{B}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}$, $\boldsymbol{E}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}$, and $\boldsymbol{H}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}$, respectively. As interpreted and used in [23], the constitutive relations described in the complex domain can greatly simplify the analytical formulation.

In physical space, the constitutive relations of the material we treat could be written as

$$\mathcal{D}(\boldsymbol{r}, t) = \bar{\varepsilon}' \cdot \mathcal{E}(\boldsymbol{r}, t) + \bar{\boldsymbol{\xi}}' \cdot \frac{\partial \mathcal{H}(\boldsymbol{r}, t)}{\partial t} \tag{1a$'$}$$

$$\mathcal{B}(\boldsymbol{r}, t) = \bar{\boldsymbol{\mu}}' \cdot \mathcal{H}(\boldsymbol{r}, t) + \bar{\boldsymbol{\mu}}'' \cdot \frac{\partial \mathcal{H}(\boldsymbol{r}, t)}{\partial t} + \bar{\boldsymbol{\zeta}}' \cdot \frac{\partial \mathcal{E}(\boldsymbol{r}, t)}{\partial t} \tag{1b$'$}$$

where

$$\bar{\varepsilon}' = \bar{\varepsilon} \tag{2a$'$}$$

$$\bar{\boldsymbol{\mu}}' = \mu_t \bar{\boldsymbol{I}}_t + \mu_z \boldsymbol{e}_z \boldsymbol{e}_z \qquad \bar{\boldsymbol{\mu}}'' = -g' \boldsymbol{e}_z \times \bar{\boldsymbol{I}}_t \tag{2b$'$}$$

$$\bar{\boldsymbol{\xi}}' = (\mu_0 \varepsilon_0)^{1/2} (-\alpha' \bar{\boldsymbol{I}}_t + \beta' \boldsymbol{e}_z \times \bar{\boldsymbol{I}}_t - \gamma' \boldsymbol{e}_z \boldsymbol{e}_z) \tag{2c$'$}$$

$$\bar{\boldsymbol{\zeta}}' = (\mu_0 \varepsilon_0)^{1/2} (\alpha' \bar{\boldsymbol{I}}_t + \beta' \boldsymbol{e}_z \times \bar{\boldsymbol{I}}_t + \gamma' \boldsymbol{e}_z \boldsymbol{e}_z) \tag{2d$'$}$$

for time harmonic $e^{i\omega t}$ time dependence of the fields, $g' = g/\omega$, $\alpha' = \alpha/\omega$, $\beta' = \beta/\omega$, and $\gamma' = \gamma/\omega$.

To have an idea of a medium with constitutive dyadics of the above forms, we first note that the special case with $g = \beta = \gamma = 0$, corresponds to the transversely chiral uniaxial bianisotropic medium studied earlier [28]. This medium can be created by suspending metal helices in a host dielectric in such a way that the axes of all helices are perpendicular to the z-axis, but possess arbitrary orientations and locations. In another special case with $g = \alpha = \gamma = 0$, the present medium becomes the uniaxial omega medium [29], which may be fabricated by immersing two ensembles of orthogonally positioned Ω-shaped particles in a host isotropic medium. When $g = \alpha = \beta = 0$, the medium is called a uniaxial chiral medium [30], which can be realized by mixing randomly oriented conductive helices with an isotropic base medium in such a manner that the axes of all helices are parallel to the z-axis. The medium under consideration reduces to a uniaxial chiro-omega medium, as γ and g vanish [31]. Uniaxial chiro-omega medium, fabricated by immersing both metal helices and Ω-shaped elements in the same host isotropic medium in a certain manner, may find applications in designing antireflection coatings and antenna radomes.

Since the constitutive relations (1) and (2) recover the cases of transversely chiral uniaxial bianisotropic medium [28], uniaxial omega medium [29], uniaxial chiral medium [30], and uniaxial chiro-omega medium [31], it is reasonable to consider that these constitutive relations could characterize the material we will try to treat in the complex domain.

The uniaxial bianisotropic-ferrite medium is a subset of the wider class referred to as bianisotropic media. Important research on general bianisotropic media have been presented by Post [32], Kong [26], and Chen [33] among others. In contradistinction to these general considerations, the present contribution is intended to derive the eigenfunction expansion of the Green dyadics in a homogeneous uniaxial bianisotropic-ferrite medium in terms of the cylindrical vector wavefunctions. Based on the completeness property of the spectral eigenwaves in the Fourier transformation spectral domain, the present formulations are considerably simplified by analytically evaluating the integrals, with respect to the spectral longitudinal and radial wavenumbers. respectively. This extended method, which is standard and straightforward, leads to two sets of the eigenfunction expansion of the Green dyadics in an unbounded uniaxial bianisotropic-ferrite medium by the cylindrical vector wavefunctions. The analysis indicates that the solutions of the source-incorporated Maxwell's equations for a uniaxial bianisotropic-ferrite medium are composed of two (or four) eigenwaves travelling with different wavenumbers. Each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave. It is also found that the Sommerfeld–Weyl-type integrals of dipole radiation in a uniaxial bianisotropic-ferrite medium involve only those Sommerfeld–Weyl-type integrals of dipole radiation in an isotropic medium. The present formulations can be used to construct the Green dyadics of planarly and cylindrically multilayered structures consisting of uniaxial bianisotropic-ferrite media, by employing the method of scattering superpositions [4, 24, 25]. The greatest advantage of the eigenfunction expansion of the Green dyadics, as presented here, is that it provides fundamental insight into the physical process of the uniaxial bianisotropic-ferrite medium, and lays the foundation to study the source-incorporated electromagnetic phenomena involving uniaxial bianisotropic-ferrite media (e.g. Raman and fluorescent scattering by active molecules embedded in a uniaxial bianisotropic-ferrite medium).

A closed-form expression of the Green dyadic for a special class of uniaxial bianisotropic media with $g = \alpha = \beta = 0$, was derived first for the reciprocal case [34], and later for the nonreciprocal case $g = 0$, and $\boldsymbol{e}_z \cdot \bar{\boldsymbol{\xi}} \cdot \boldsymbol{e}_z \neq -\boldsymbol{e}_z \cdot \bar{\boldsymbol{\zeta}} \cdot \boldsymbol{e}_z$ [35]. In [36], a rigorous investigation was presented by Weiglhofer for the possibility of deriving the closed-form representations of the Green dyadics in a general uniaxial media. In that paper, it was shown that at least one of three possible relations among the constitutive parameters has to be satisfied to allow the closed-form solution of the Green dyadics. It was also pointed out, however, that these relations are only necessary relations and not sufficient relations to allow the closed-form solution. The most important of these three cases is the case with $g = \beta = \gamma = 0$, for which the closed-form solutions of the Green dyadics were presented in [36]. Most recently, Olyslager [37] presented the closed-form representations of the Green dyadics for a uniaxial bianisotropic media with $g = \beta = 0$. In view of the uniformity constraint condition for the uniaxial bianisotropic media [27], the materials treated in [34–37] are just the special cases of the media studied here. The methods used by the authors of [34–37] do not seem to be applicable for the present uniaxial bianisotropic-ferrite media to allow the closed-form representations of the Green dyadics. Moreover, the Green dyadics represented in the forms of the eigenfunction expansion seem to be more important and attractive than those expressed in the closed forms in practical applications (e.g. to study Raman and fluorescent scattering by active molecules embedded in the given complex media, to establish T-matrix formulation for the electromagnetic boundary value problems involving complex media, and to take an insight into the physical process of the material under construction).

2. Eigenwaves in uniaxial bianisotropic-ferrite medium

Substituting the constitutive relations (1(*a*)) and (1(*b*)) into the source-incorporated Maxwell's equations, a compact form of the field equations in the uniaxial bianisotropic-ferrite medium is obtained

$$\begin{pmatrix} \omega\bar{\varepsilon} & \omega\bar{\boldsymbol{\xi}} + \mathrm{i}\nabla\times \\ \omega\bar{\boldsymbol{\zeta}} - \mathrm{i}\nabla\times & \omega\bar{\boldsymbol{\mu}} \end{pmatrix} \begin{pmatrix} \boldsymbol{E}(\boldsymbol{r}) \\ \boldsymbol{H}(\boldsymbol{r}) \end{pmatrix} = \begin{pmatrix} \mathrm{i}\boldsymbol{J}(\boldsymbol{r}) \\ \mathrm{i}\boldsymbol{M}(\boldsymbol{r}) \end{pmatrix} \tag{3}$$

where $\boldsymbol{J}$ and $\boldsymbol{M}$ denote the electric and magnetic exciting currents, respectively.

To examine the physical properties of the eigenwaves in the uniaxial bianisotropic-ferrite medium, Fourier transformation for the electromagnetic fields and exciting sources is introduced:

$$\boldsymbol{F}(\boldsymbol{r}) = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \boldsymbol{F}(\boldsymbol{k}) e^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \,\mathrm{d}\boldsymbol{k} \tag{4}$$

where $\boldsymbol{F} = \boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{J}$, or $\boldsymbol{M}$, and $\boldsymbol{k} = k_x\boldsymbol{e}_x + k_y\boldsymbol{e}_y + k_z\boldsymbol{e}_z$. Then, (3) can be rewritten in the Fourier spectral domain

$$\begin{pmatrix} \omega\bar{\varepsilon} & \omega\bar{\boldsymbol{\xi}} + \boldsymbol{k}\times \\ \omega\bar{\boldsymbol{\zeta}} - \boldsymbol{k}\times & \omega\bar{\boldsymbol{\mu}} \end{pmatrix} \begin{pmatrix} \boldsymbol{E}(\boldsymbol{k}) \\ \boldsymbol{H}(\boldsymbol{k}) \end{pmatrix} = \begin{pmatrix} \mathrm{i}\boldsymbol{J}(\boldsymbol{k}) \\ \mathrm{i}\boldsymbol{M}(\boldsymbol{k}) \end{pmatrix}. \tag{5}$$

For the sake of brevity, (5) is denoted as

$$\bar{\boldsymbol{L}} \cdot \boldsymbol{\Psi}(\boldsymbol{k}) = \boldsymbol{\Phi}(\boldsymbol{k}) \tag{6}$$

where $\bar{\boldsymbol{L}}$ is a Hermitian operator ($\bar{\boldsymbol{L}} = \bar{\boldsymbol{L}}^{*T}$, where the superscripts asterisk and T represent the complex conjugate and transpose operators, respectively). Here,

$$\boldsymbol{\Psi}(\boldsymbol{k}) = [\boldsymbol{E}(\boldsymbol{k}), \boldsymbol{H}(\boldsymbol{k})]^T \qquad \text{and} \qquad \boldsymbol{\Phi}(\boldsymbol{k}) = [\mathrm{i}\boldsymbol{J}(\boldsymbol{k}), \mathrm{i}\boldsymbol{M}(\boldsymbol{k})]^T.$$

The characteristic equation, which determines the wavenumbers of the eigenwaves propagating in the uniaxial bianisotropic-ferrite medium, can be straightforwardly obtained by requiring the determinant of operator $\bar{\boldsymbol{L}}$ be zero. Algebraic manipulation results in

$$\begin{aligned} &\varepsilon'(f^2 - a)k_\rho^4 + [(k_z^2 - a)(e^2 + f^2 - a - \varepsilon' a') + (bk_z + c)(bk_z - 2ek_z + c)]k_\rho^2 \\ &\quad -[(k_z^2 - a)^2 + (bk_z + c)^2]a' = 0 \end{aligned} \tag{7}$$

where $k_\rho = (k_x^2 + k_y^2)^{1/2}$, and

$$\begin{aligned} a &= \omega^2[\varepsilon_t\mu_t - \varepsilon_0\mu_0(\alpha^2 + \beta^2)] \\ b &= 2\mathrm{i}k_0\alpha \\ c &= \mathrm{i}\omega^2 g \\ e &= \mathrm{i}k_0(\alpha + \gamma\varepsilon') \\ f &= \mathrm{i}k_0\beta \\ \varepsilon' &= \varepsilon_t/\varepsilon_z \\ a' &= \omega^2(\varepsilon_t\mu_z - \varepsilon_0\mu_0\gamma^2\varepsilon'). \end{aligned} \tag{8}$$

It is obvious that the characteristic equation (7) is an even function of k_ρ. We can regard this characteristic equation (7) as a function of k_ρ (or k_z), where k_ρ (or k_z) is determined by k_z (or k_ρ). The roots of (7) are designed as $k_\rho = k_{\rho q}$ (or $k_z = k_{zq}$), where $q = 1, 2, 3$ and 4. It is worthy to note the important property of the roots of (7): $k_{\rho q}$ (or k_{zq}) is independent of ϕ_k, with $\phi_k = tg^{-1}(k_y/k_x)$.

The eigenwaves corresponding to the qth root of (7), expressed in a circular cylindrical coordinate system, can be derived by substituting either $k_\rho = k_{\rho q}$ or $k_z = k_{zq}$ in the following expression

$$\boldsymbol{\Psi}_q^\sigma(\boldsymbol{k}) = \begin{pmatrix} E_{q\rho}^\sigma(\boldsymbol{k}) \\ E_{q\phi}^\sigma(\boldsymbol{k}) \\ E_{qz}^\sigma(\boldsymbol{k}) \\ H_{q\rho}^\sigma(\boldsymbol{k}) \\ H_{q\phi}^\sigma(\boldsymbol{k}) \\ H_{qz}^\sigma(\boldsymbol{k}) \end{pmatrix} \begin{pmatrix} C_q^\sigma(k_\rho, k_z)\cos(\phi - \phi_k) + D_q^\sigma(k_\rho, k_z)\sin(\phi - \phi_k) \\ -C_q^\sigma(k_\rho, k_z)\sin(\phi - \phi_k) + D_q^\sigma(k_\rho, k_z)\cos(\phi - \phi_k) \\ \frac{1}{\omega\varepsilon_z}[\mathrm{i}k_0\gamma - k_\rho B_q^\sigma(k_\rho, k_z)] \\ A_q^\sigma(k_\rho, k_z)\cos(\phi - \phi_k) + B_q^\sigma(k_\rho, k_z)\sin(\phi - \phi_k) \\ -A_q^\sigma(k_\rho, k_z)\sin(\phi - \phi_k) + D_q^\sigma(k_\rho, k_z)\cos(\phi - \phi_k) \\ 1 \end{pmatrix} \tag{9}$$

with $\phi = tg^{-1}(y/x)$, $\sigma = \rho$ for $k_\rho = k_{\rho q}$, and $\sigma = z$ for $k_z = k_{zq}$. Here, the spectral parameters are explicitly presented in appendix A.

To reveal the biorthogonality property of these eigenvalues, equation (6) should be rewritten in other forms. First, regarding $k_{\rho q}$ as the roots of the characteristic equation (7), (6) is rewritten as

$$\bar{\boldsymbol{A}}_1 \cdot \boldsymbol{\Psi}_q^\rho(\boldsymbol{k}) - k_{\rho q}\bar{\boldsymbol{B}}_1 \cdot \boldsymbol{\Psi}_q^\rho(\boldsymbol{k}) = \boldsymbol{\Phi}(\boldsymbol{k}) \tag{10}$$

where both $\bar{A}_1$ and $\bar{B}_1$ are Hermitian operators. These eigenvalues $\Psi_q^{\rho}(k)$, which form a complete set in the spectral space [33], are biorthogonality [33, 38]: $\Psi_p^{\rho*}(k)\cdot\bar{B}_1\cdot\Psi_q^{\rho}(k) = N_q^2\delta_{pq}$. Here, δ_{pq} denotes the Kronecker delta function (i.e. it is 1 for $p = q$, and 0 for $p \neq q$), and the biorthogonality coefficient calculated as

$$N_q^2 + \frac{\mathrm{i}k_0\gamma}{\omega\varepsilon_z}\{[B_q^{\rho}(k_{\rho q}, k_z)]^* - B_q^{\rho}(k_{\rho q}, k_z)\} - D_q^{\rho}(k_{\rho q}, k_z) - [D_q^{\rho}(k_{\rho q}, k_z)]^* \\ - \frac{2k_{\rho q}}{\omega\varepsilon_z}[B_q^{\rho}(k_{\rho q}, k_z)][B_q^{\rho}(k_{\rho q}, k_z)]^*.$$

An alternative useful rewritten form of (6) is

$$\bar{A}_2\cdot\Psi_q^z(k) - k_{zq}\bar{B}_2\cdot\Psi_q^z(k) = \Phi(k) \qquad (11)$$

where both $\bar{A}_2$ and $\bar{B}_2$ are Hermitian operators, and the roots of the characteristic equation (7) are considered to be k_{zq}. The eigenwaves of (11), which form a complete set in the spectral space [33], are also biorthogonality [33, 38]: $\Psi_p^{z*}(k)\cdot\bar{B}_2\cdot\Psi_q^z(k) = M_q^2\delta_{pq}$. Here, the biorthogonality coefficient is found to be

$$M_q^2 = [A_q^z(k_\rho, k_{zq})][D_q^z(k_\rho, k_{zq})]^* + [D_q^z(k_\rho, k_{zq})][A_q^z(k_\rho, k_{zq})]^* \\ -[B_q^z(k_\rho, k_{zq})][C_q^z(k_\rho, k_{zq})]^* - [C_q^z(k_\rho, k_{zq})][B_q^z(k_\rho, k_{zq})]^*.$$

Based on the completeness properties of the above-presented eigenwaves $\Psi_q^z(k)$ and $\Psi_q^{\rho}(k)$, the solutions of the spectral source-incorporated equation (6) can be represented in terms of these eigenwaves [2, 33, 38]

$$\Psi(k) = \sum_q \frac{\Psi_q^z(k)\Psi_q^{z*}(k)}{(k_{zq} - k_z)M_q^2}\cdot\Phi(k) \qquad (12)$$

or

$$\Psi(k) = \sum_q \frac{\Psi_q^{\rho}(k)\Psi_q^{\rho*}(k)}{(k_{\rho q} - k_\rho)N_q^2}\cdot\Phi(k). \qquad (13)$$

In this way, the solutions of the spectral source-incorporated Maxwell's equation (5) are represented in terms of the spectral eigenwaves in the uniaxial bianisotropic-ferrite medium. These expressions, (12) and (13), are our starting point in constructing the eigenfunction expansion of the Green dyadics, as will be reported in detail in the following analysis.

3. Green dyadics in unbounded uniaxial bianisotropic-ferrite medium

For the sake of simplicity, we define the Green dyadics $\bar{G}(r, r')$ in the homogeneous uniaxial bianisotropic-ferrite medium as

$$\begin{pmatrix} E(r) \\ H(r) \end{pmatrix} = \int_{V'} \bar{G}(r, r')\cdot\begin{pmatrix} \mathrm{i}J(r') \\ \mathrm{i}M(r') \end{pmatrix}\mathrm{d}V', \qquad (14)$$

where V' is the volume occupied by the electric and magnetic exciting currents. The definition (14) indicates that the electromagnetic fields associated with the current sources can be expressed as a convolution of the current distribution and the three-dimensional free-space Green dyadics.

Using the definition of Green dyadics (14) and equations (12) and (13), the Green dyadics in the uniaxial bianisotropic-ferrite medium can be represented in terms of the corresponding spectral eigenwaves

$$\bar{G}(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{k} \sum_q \frac{\boldsymbol{\Psi}_q^z(\boldsymbol{k}) \boldsymbol{\Psi}_q^{z*}(\boldsymbol{k})}{(k_{zq} - k_z) M_q^2} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \tag{15}$$

$$\bar{G}(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{8\pi^3} \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{k} \sum_q \frac{\boldsymbol{\Psi}_q^\rho(\boldsymbol{k}) \boldsymbol{\Psi}_q^{\rho*}(\boldsymbol{k})}{(k_{\rho q} - k_p) N_q^2} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}. \tag{16}$$

Here, the convolution theorem of Fourier transformation has been employed.

It is helpful to mention that equation (15) is suitable to construct the Green dyadics of planarly multilayered uniaxial bianisotropic-ferrite media, while equation (16) is a useful tool to formulate the Green dyadics of a cylindrical multilayered structure consisting of uniaxial bianisotropic-ferrite media.

To represent the Green dyadics in the forms of the eigenfunction expansion in terms of the cylindrical vector wavefunctions, integrals with respect to the spectral longitudinal and radial wavenumbers in equations (15) and (16) respectively will be evaluated analytically.

3.1. Analytical evaluation of the integral with respect to the spectral longitudinal wavenumber

In this subsection, we will try to represent (15) in the form of the eigenfunction expansion in terms of the cylindrical vector wavefunctions. For this purpose, the integral with respect to the spectral longitudinal wavenumber, k_z, is analytically evaluated by using the residue method, which results in

$$\begin{aligned}\bar{G}(\boldsymbol{r}, \boldsymbol{r}') &= \begin{pmatrix} \bar{G}_{ee}(\boldsymbol{r}, \boldsymbol{r}') & \bar{G}_{em}(\boldsymbol{r}, \boldsymbol{r}') \\ \bar{G}_{me}(\boldsymbol{r}, \boldsymbol{r}') & \bar{G}_{mm}(\boldsymbol{r}, \boldsymbol{r}') \end{pmatrix} \\ &= \frac{\mathrm{i}}{8\pi^2} \int_0^\infty \mathrm{d}k_\rho \int_{\phi_k=0}^{2\pi} \mathrm{d}\phi_k \sum_{q=1}^4 \frac{\boldsymbol{\Psi}_q^z(\boldsymbol{k}) \boldsymbol{\Psi}_q^{z*}(\boldsymbol{k})}{M_q^2} \mathrm{e}^{-\mathrm{i}k_{zq}(z-z')} \mathrm{e}^{-\mathrm{i}k_\rho \rho \cos(\phi-\phi_k)} \mathrm{e}^{\mathrm{i}k_\rho \rho' \cos(\phi'-\phi_k)}\end{aligned} \tag{17}$$

where $\rho = (x^2 + y^2)^{1/2}$. Here, the 3×3 dyadics $\bar{G}_{ee}(\boldsymbol{r}, \boldsymbol{r}')$ and $\bar{G}_{mm}(\boldsymbol{r}, \boldsymbol{r}')$ are the Green dyadics of electric and magnetic types, respectively; while $\bar{G}_{em}(\boldsymbol{r}, \boldsymbol{r}')$ and $\bar{G}_{me}(\boldsymbol{r}, \boldsymbol{r}')$ are the pseudo-type Green dyadics. It should be recognized that the following formulations are essentially based on the fact that the spectral longitudinal wavenumber k_{zq} is independent of the spectral azimuthal angle ϕ_k.

Substituting into (17) the explicit expression of $\boldsymbol{\Psi}_q^z(\boldsymbol{k})$ and the well known identities

$$\mathrm{e}^{-\mathrm{i}k_\rho \rho \cos(\phi-\phi_k)} = \sum_{n=-\infty}^{\infty} (-\mathrm{i})^n J_n(k_\rho \rho) \mathrm{e}^{-\mathrm{i}n(\phi-\phi_k)} \tag{18}$$

$$\mathrm{e}^{\mathrm{i}k_\rho \rho' \cos(\phi'-\phi_k)} = \sum_{m=-\infty}^{\infty} (\mathrm{i})^m J_m(k_\rho \rho') \mathrm{e}^{\mathrm{i}m(\phi'-\phi_k)} \tag{19}$$

after cumbersome mathematical manipulation by properly grouping the terms involving the integrals for the ϕ_k variable and introducing the cylindrical vector wavefunctions, we end up with

$$\begin{aligned}\bar{G}_{mm}(\boldsymbol{r}, \boldsymbol{r}') = \frac{\mathrm{i}}{8\pi} \int_0^\infty \mathrm{d}k_\rho \sum_{q=1}^4 \frac{1}{M_q^2} \sum_{n=-\infty}^{\infty} (-1)^n [a_q^z(k_\rho, k_{zq}) \boldsymbol{M}_n^{(1)}(k_\rho, k_{zq}) \\ + b_q^z(k_\rho, k_{zq}) \boldsymbol{N}_n^{(1)}(k_\rho, k_{zq}) + c_q^z(k_\rho, k_{zq}) \boldsymbol{L}_n^{(1)}(k_\rho, k_{zq})]\end{aligned}$$

$$\times[a_q^{z'}(k_\rho, k_{zq})\boldsymbol{M}_{-n}^{(1)'}(k_\rho, -k_{zq}) + b_q^{'z}(k_\rho, k_{zq})\boldsymbol{N}_{-n}^{(1)'}(k_\rho, -k_{zq})$$
$$+c_q^{z'}(k_\rho, k_{zq})\boldsymbol{L}_{-n}^{(1)'}(k_{\rho'}, -k_{zq})] \tag{20}$$

where the primes over the vector wavefunctions denote that they are evaluated at $\boldsymbol{r}'$. Here, the techniques of mathematical manipulation are similar to those we have used in [9–12] to obtain the field representations in the source-free regions. The expansion coefficients are found to be

$$a_q^\sigma(k_\rho, k_z) = -\frac{2\mathrm{i}B_q^\sigma(k_\rho, k_z)}{k_\rho} \tag{21}$$

$$b_q^\sigma(k_\rho, k_z) = -\frac{2k_q A_q^\sigma(k_\rho, k_z)}{k_\rho k_z} + \frac{2}{k_q}\left[1 + \frac{k_\rho A_q^\sigma(k_\rho, k_z)}{k_z}\right] \tag{22}$$

$$c_q^\sigma(k_\rho, k_z) = \frac{2\mathrm{i}k_z}{k_q^2}\left[1 + \frac{k_\rho A_q^\sigma(k_\rho, k_z)}{k_z}\right] \tag{23}$$

with $k_q = (k_z^2 + k_\rho^2)^{1/2}$. $a_q^{\sigma'}(k_\rho, k_z)$, $b_q^{\sigma'}(k_\rho, k_z)$ and $c_q^{\sigma'}(k_\rho, k_z)$ are separately derived from $a_q^\sigma(k_\rho, k_z)$, $b_q^\sigma(k_\rho, k_z)$ and $c_q^\sigma(k_\rho, k_z)$ with the replacement of $A_q^\sigma(k_\rho, k_z)$ and $B_q^\sigma(k_\rho, k_z)$ by their complex conjugates, respectively. It should be noted that the roots $k_z = k_{zq}$ or (7) are chosen such that $\mathrm{Re}[k_{zq}] > 0$ for $z > z'$, and $\mathrm{Re}[k_{zq}] < 0$ for $z < z'$, where Re[.] denotes the real part of a complex function. Here, the cylindrical vector wavefunctions are defined in appendix B.

The Green dyadic of electric type $\bar{\boldsymbol{G}}_{ee}(\boldsymbol{r}, \boldsymbol{r}')$ can be obtained from $\bar{\boldsymbol{G}}_{mm}(\boldsymbol{r}, \boldsymbol{r}')$, with the replacement of a_q^z, b_q^z, c_q^z, $a_q^{z'}$, $b_q^{z'}$, $c_q^{z'}$ by d_q^z, e_q^z, f_q^z, $d_q^{z'}$, $e_q^{z'}$, $f_q^{z'}$, respectively. Here, the expansion coefficients are determined as

$$d_q^\sigma(k_\rho, k_z) = -\frac{2\mathrm{i}D_q^\sigma(k_\rho, k_z)}{k_\rho} \tag{24}$$

$$e_q^\sigma(k_\rho, k_z) = -\frac{2k_z C_q^\sigma(k_\rho, k_z)}{k_\rho, k_q} + \frac{2[\mathrm{i}k_0\gamma - k_\rho B_q^\sigma(k_\rho, k_z)]}{k_q\omega\varepsilon_z} \tag{25}$$

$$f_q^\sigma(k_\rho, k_z) = \frac{2\mathrm{i}}{k_q^2}\left[k_\rho C_q^\sigma(k_\rho, k_z) + \frac{k_z[\mathrm{i}k_0\gamma - k_\rho B_q^\sigma(k_\rho, k_z)]}{\omega\varepsilon_z}\right] \tag{26}$$

for $\sigma = z$, and $k_z = k_{zq}$. $d_q^{\sigma'}(k_\rho, k_z)$, $e_q^{\sigma'}(k_\rho, k_z)$ and $f_q^{\sigma'}(k_\rho, k_z)$ can be obtained from $d_q^\sigma(k_\rho, k_z)$, $e_q^\sigma(k_\rho, k_z)$ and $f_q^\sigma(k_\rho, k_z)$ with the replacement of $C_q^\sigma(k_\rho, k_z)$, $D_q^\sigma(k_\rho, k_z)$ and $[\mathrm{i}k_0\gamma - k_\rho B_q^\sigma(k_\rho, k_z)]$ by their complex conjugates, respectively. The pseudo-type Green dyadics $\bar{\boldsymbol{G}}_{em}(\boldsymbol{r}, \boldsymbol{r}')$ can be obtained from $\bar{\boldsymbol{G}}_{mm}(\boldsymbol{r}, \boldsymbol{r}')$, with the substitution of a_q^z, b_q^z, c_q^z by d_q^z, e_q^z, f_q^z, respectively; and $\bar{\boldsymbol{G}}_{me}(\boldsymbol{r}, \boldsymbol{r}')$ can be derived from $\bar{\boldsymbol{G}}_{mm}(\boldsymbol{r}, \boldsymbol{r}')$, with the replacement of $a_q^{z'}$, $b_q^{z'}$, $c_q^{z'}$ by $d_q^{z'}$, $e_q^{z'}$, $f_q^{z'}$, separately.

It should be mentioned that the present eigenfunction expansion of the Green dyadics can be reduced to the counterparts of the reciprocal chiral medium [24], if letting $\varepsilon_t = \varepsilon_z = \varepsilon$, $\mu_t = \mu_z = \mu$, $\alpha = \gamma = \xi_c$, and $g = \beta = 0$ in the constitutive relations. This set of the eigenfunction representation of the Green dyadics can be used to construct the Green dyadics of planarly multilayered uniaxial bianisotropic-ferrite media, by applying the method of scattering superposition [4, 24] and appropriate electromagnetic boundary conditions.

Straightforward mathematical analysis reveals that for dipole sources parallel to the z-axis, only the terms corresponding to $n = 0$ exist for the Green dyadics, while the Green dyadics of dipole sources perpendicular to the z-axis contain only the $n = 1$ terms. Therefore, Sommerfeld–Weyl-type integrals of dipole radiation in a uniaxial bianisotropic-ferrite medium involve only those Sommerfeld–Weyl-type integrals of dipole radiation in an isotropic medium [39]. So, various approximate, asymptotic, and numerical methods for Sommerfeld–Weyl-type integrals [39] can be applied to study the electromagnetic resonance, radiation, propagation, and scattering phenomena of planarly multilayered uniaxial bianisotropic-ferrite media.

3.2. Analytical evaluation of the integral with respect to the spectral radial wavenumber

In this subsection, we will try to represent (16) in the form of the eigenfunction expansion in terms of the cylindrical vector wavefunctions. To this end, employing the identities (18) and (19), the integral with respect to the spectral radial wavenumber k_ρ is analytically evaluated by applying the residue calculus through a modified contour in the k_ρ plane, which results in

$$\bar{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}') = \begin{pmatrix} \bar{\boldsymbol{G}}_{ee}(\boldsymbol{r},\boldsymbol{r}') & \bar{\boldsymbol{G}}_{em}(\boldsymbol{r},\boldsymbol{r}') \\ \bar{\boldsymbol{G}}_{me}(\boldsymbol{r},\boldsymbol{r}') & \bar{\boldsymbol{G}}_{mm}(\boldsymbol{r},\boldsymbol{r}') \end{pmatrix}$$

$$= \begin{cases} \dfrac{\mathrm{i}}{16\pi^2}\displaystyle\int_{-\infty}^{\infty}\mathrm{d}k_z\int_{\phi_k=0}^{2\pi}\mathrm{d}\phi_k\sum_q\frac{\boldsymbol{\Psi}_q^{\rho}(\boldsymbol{k})\boldsymbol{\Psi}_q^{\rho*}(\boldsymbol{k})}{N_q^2}\mathrm{e}^{-\mathrm{i}k_z(z-z')} \\ \times\displaystyle\sum_{n=-\infty}^{\infty}(-\mathrm{i})^n J_n(k_{\rho q}\rho)\mathrm{e}^{-\mathrm{i}n(\phi-\phi_k)}\sum_{m=-\infty}^{\infty}\mathrm{i}^m H_m^{(2)}(k_{\rho q}\rho')\mathrm{e}^{-\mathrm{i}m(\phi'-\phi_k)} & \rho\leqslant\rho' \\ \dfrac{\mathrm{i}}{16\pi^2}\displaystyle\int_{-\infty}^{\infty}\mathrm{d}k_z\int_{\phi_k=0}^{2\pi}\mathrm{d}\phi_k\sum_q\frac{\boldsymbol{\Psi}_q^{\rho}(\boldsymbol{k})\boldsymbol{\Psi}_q^{\rho*}(\boldsymbol{k})}{N_q^2}\mathrm{e}^{-\mathrm{i}k_z(z-z')} \\ \times\displaystyle\sum_{n=-\infty}^{\infty}(-\mathrm{i})^n H_n^{(2)}(k_{\rho q}\rho)\mathrm{e}^{-\mathrm{i}n(\phi-\phi_k)}\sum_{m=-\infty}^{\infty}\mathrm{i}^m J_m(k_{\rho q}\rho')\mathrm{e}^{\mathrm{i}m(\phi'-\phi_k)} & \rho\geqslant\rho'. \end{cases} \tag{27}$$

Here, we have employed the identity [4]

$$\int_0^{\infty}\mathrm{d}k_\rho\frac{\bar{\boldsymbol{T}}[J_n(k_\rho\rho)J_n(k_\rho\rho')]}{(k_{\rho q}-k_\rho)N_q^2} = \frac{\mathrm{i}\pi}{2N_q^2}\bar{\boldsymbol{T}}[H_n^{(2)}(k_{\rho q}\rho_>)J_n(k_{\rho q}\rho_<)] \tag{28}$$

where $\rho_> = \max(\rho,\rho')$, $\rho_< = \min(\rho,\rho')$, and $\bar{\boldsymbol{T}}$ stands for a dyadic operator, having the property of $\bar{\boldsymbol{T}}(k_\rho) = -\bar{\boldsymbol{T}}(-k_\rho)$.

Substituting the explicit expression of $\boldsymbol{\Psi}_q^{\rho}(\boldsymbol{k})$ in (27) and properly grouping the terms involving the integrals with respect to the ϕ_k variable, the Green dyadics in the uniaxial bianisotropic-ferrite medium can be represented in terms of the cylindrical vector wavefunctions:

$$\begin{aligned}\bar{\boldsymbol{G}}_{mm}(\boldsymbol{r},\boldsymbol{r}') = {}& \frac{\mathrm{i}}{16\pi}\int_{-\infty}^{\infty}\mathrm{d}k_z\sum_{q=1}^{2}\frac{1}{N_q^2}\sum_{n=-\infty}^{\infty}(-1)^n[a_q^{\rho}(k_{\rho q},k_z)\boldsymbol{M}_n^{(\tau_1)}(k_{\rho q},k_z) \\ &+b_q^{\rho}(k_{\rho q},k_z)\boldsymbol{N}_n^{(\tau_1)}(k_{\rho q},k_z)+c_q^{\rho}(k_{\rho q},k_z)\boldsymbol{L}_n^{(\tau_1)}(k_{\rho q},k_z)] \\ &\times[a_q^{\rho'}(k_{\rho q},k_z)\boldsymbol{M}_{-n}^{(\tau_2)'}(k_{\rho q},-k_z)+b_q^{\rho'}(k_{\rho q},k_z)\boldsymbol{N}_{-n}^{(\tau_2)'}(k_{\rho q},-k_z) \\ &+c_q^{\rho'}(k_{\rho q},k_z)\boldsymbol{L}_{-n}^{(\tau_2)'}(k_{\rho q},-k_z)]\end{aligned} \tag{29}$$

$\bar{\boldsymbol{G}}_{ee}(\boldsymbol{r},\boldsymbol{r}')$ can be obtained from $\bar{\boldsymbol{G}}_{mm}(\boldsymbol{r},\boldsymbol{r}')$, with the replacement of a_q^{ρ}, b_q^{ρ}, c_q^{ρ}, $a_q^{\rho'}$, $b_q^{\rho'}$, $c_q^{\rho'}$ by d_q^{ρ}, e_q^{ρ}, f_q^{ρ}, $d_q^{\rho'}$, $e_q^{\rho'}$, $f_q^{\rho'}$, respectively; $\bar{\boldsymbol{G}}_{em}(\boldsymbol{r},\boldsymbol{r}')$ can be derived from $\bar{\boldsymbol{G}}_{mm}(\boldsymbol{r},\boldsymbol{r}')$, with separate substitution of a_q^{ρ}, b_q^{ρ}, c_q^{ρ} by d_q^{ρ}, e_q^{ρ}, f_q^{ρ}; $\bar{\boldsymbol{G}}_{me}(\boldsymbol{r},\boldsymbol{r}')$ can be obtained from $\bar{\boldsymbol{G}}_{mm}(\boldsymbol{r},\boldsymbol{r}')$, with the replacement of $a_q^{\rho'}$, $b_q^{\rho'}$, $c_q^{\rho'}$ by $d_q^{\rho'}$, $e_q^{\rho'}$, $f_q^{\rho'}$, respectively. Here, $\tau_1 = 1$,

$\tau_2 = 4$ for $\rho \leqslant \rho'$, and $\tau_1 = 4$, $\tau_2 = 1$ for $\rho \geqslant \rho'$. The expansion coefficients used here can be straightforwardly obtained from equations (21–23) and (24–26), with the substitution of $\sigma = \rho$ and $k_\rho = k_{\rho q}$.

In equation (29), $k_{\rho 3}$ and $k_{\rho 4}$ are not included in the summation since $k_{\rho 3} = -k_{\rho 1}$, $k_{\rho 4} = -k_{\rho 2}$ and these symmetric roots are automatically taken into account as the spectral azimuthal angle ϕ_k spans from 0 to 2π.

It should be pointed out that the Green dyadics represented in the form of the eigenfunction expansion, as given in this subsection, can be verified by comparing their special forms with the counterparts of reciprocal chiral medium [25] and isotropic medium [4]. Moreover, they can be used to construct the Green dyadics of a cylindrically multilayered structure consisting of uniaxial bianisotropic-ferrite media, by employing the method of scattering superposition [4, 25] and appropriate electromagnetic boundary conditions.

The resulting equations in this subsection indicate that the electromagnetic waves in an unbounded uniaxial bianisotropic-ferrite medium are transversely outgoing for $\rho \geqslant \rho'$, and transversely standing for $\rho \leqslant \rho'$. This physical property of the electromagnetic waves is similar to that of a dielectric leaky antenna with infinitely long circular cylindrical structure, positioned in an unbounded isotropic medium.

From the present formulations, it is easily seen that $\bar{\boldsymbol{G}}_{RS}(\boldsymbol{r}, \boldsymbol{r}') \not\equiv \bar{\boldsymbol{G}}^T_{RS}(\boldsymbol{r}', \boldsymbol{r})$ with $R(S) = e$ or m, as can also be directly obtained from the reciprocal theorem [26]. In addition, it can be straightforward to derive the mathematical relationship among these Green dyadics, which could also be obtained from the definition of the Green dyadics (14) and the source-incorporated Maxwell's equations (3):

$$\bar{\boldsymbol{G}}_{em}(\boldsymbol{r}, \boldsymbol{r}') = -\frac{\mathrm{i}}{\omega}\bar{\varepsilon}^{-1} \cdot [(\nabla \times \bar{\boldsymbol{I}} - \mathrm{i}\omega\bar{\boldsymbol{\xi}}) \cdot \bar{\boldsymbol{G}}_{mm}(\boldsymbol{r}, \boldsymbol{r}')] \tag{30}$$

$$\bar{\boldsymbol{G}}_{em}(\boldsymbol{r}, \boldsymbol{r}') = \frac{\mathrm{i}}{\omega}\bar{\boldsymbol{\mu}}^{-1} \cdot [(\nabla \times \bar{\boldsymbol{I}} + \mathrm{i}\omega\bar{\boldsymbol{\zeta}}) \cdot \bar{\boldsymbol{G}}_{ee}(\boldsymbol{r}, \boldsymbol{r}')] \tag{31}$$

where $\bar{\boldsymbol{I}}$ denotes the 3×3 unit dyadic.

The electromagnetic fields associated with the exciting sources can be obtained from (14), by substituting either set of the above-presented Green dyadics. From the present formulations, it can be seem that the solutions of the source-incorporated Maxwell's equations for homogeneous uniaxial bianisotropic-ferrite medium are composed of two (or four) eigenwaves travelling with different wavenumbers. Each of these eigenwaves is a superposition of two transverse waves (**M** and **N** represent two transverse waves) and a longitudinal wave.

The essential idea of the method employed here, which is standard and straightforward, can be exploited to derive the eigenfunction expansion of the Green dyadics in a spherical coordinate system. However, since the wavenumbers of the eigenwaves are functions of the direction of these eigenwaves, simple compact forms of the field representations (corresponding to those of [9–12]) in the source-free uniaxial bianisotropic-ferrite media by the spherical vector wavefunctions cannot be obtained, and the solutions of the source-incorporated Maxwell's equations cannot be directly formulated in compact forms of the spherical vector wavefunctions, either. In the circular cylindrical coordinate system, however, it is seen from the present formulations that since the wavenumbers of the eigenwaves do not depend on the spectral azimuthal angle ϕ_k, the solutions of the source-incorporated Maxwell's equations in the uniaxial bianisotropic-ferrite medium can be represented in compact forms of the cylindrical vector wavefunctions.

4. Concluding remarks

In the present contribution, the eigenfunction expansion of the Green dyadics in an unbounded uniaxial bianisotropic-ferrite medium are developed in terms of the cylindrical vector wavefunctions, based on the concept of spectral eigenwaves. The analysis indicates that the solutions of the source-incorporated Maxwell's equations in a uniaxial bianisotropic-ferrite medium are composed of two (or four) eigenwaves travelling with different wavenumbers. Each of these eigenwaves is a superposition of two transverse waves and a longitudinal wave. The Green dyadics of planarly and cylindrically multilayered structures consisting of uniaxial bianisotropic-ferrite media can be straightforwardly obtained by employing the method of scattering superposition and appropriate electromagnetic boundary conditions, respectively. The constraint condition of the present approach, which is standard and straightforward, is that the spectral longitudinal (and radial) wavenumbers do not depend on the spectral azimuthal angle ϕ_k. In spite of this constraint condition which makes the approach employed here only applicable to a limited class of materials, the present formulations can be used to analyse and understand the physical phenomena of the source-incorporated electromagnetic boundary value problems involving unbounded or multilayered uniaxial bianisotropic-ferrite media. It is of interest to note that cylindrical vector wavefunctions can be expanded as discrete sums of the spherical vector wavefunctions [40], therefore the present formulations could be extended to solve the problems of spherical structures. Since the uniaxial bianisotropic-ferrite media studied here recover the isotropic media [4, 14], uniaxial bianisotropic media [30], transversely chiral uniaxial bianisotropic media [28], uniaxial chiro-omega media [31], and the extensively studied chiral media [24, 25], the present formulations can be specifically applied to these materials, and theoretically verified by comparing their special forms with the already existing results corresponding to the isotropic media [4] and reciprocal chiral media [24, 25]. When the present uniaxial bianisotropic-ferrite media reduce to the media treated in [34–37], the Green dyadics formulated here can be represented in simple closed forms as those of [34–37], after the integrals are explicitly evaluated, respectively. In addition, the method employed here can be extended to derive the eigenfunction expansion of Green dyadics in other kinds of media, such as transversely isotropic elastic media [41], transversely isotropic piezoelectric solids [42], and transversely isotropic saturated porous media [43]. Although the present formulations are somewhat cumbersome which is inevitable due to the complexity of the material we have tried to tackle, they are important and useful in analysing and understanding the (equivalently) source-incorporated electromagnetic phenomena of the uniaxial bianisotropic-ferrite media. Even if there exist two operations in the present formulations which are over infinite domains, convergence of these operations has been numerically examined for the source-free problems [9–12]. For the source-incorporated problems, verification for the convergence of the operations is straightforward. Moreover, these two operations over infinite domains also exist for isotropic media [4] and reciprocal chiral media [24, 25], therefore various numerical and asymptotic methods [39] can be employed to simplify the computation in practical applications. In our previous investigation, the problems we treated are the source-free Maxwell's equations in the given complex media [9–12], while in the present study we have tried to analytically solve the source-incorporated problems by the cylindrical vector wavefunctions. From a mathematical point of view, our previous investigation [9–12] is essentially based on the method of spectral angular expansion, while the present starting point is the completeness property of the spectral eigenwaves. It is believed that the present formulations provide fundamental basis to analyse the (equivalently) source-incorporated electromagnetic phenomena of uniaxial bianisotropic-ferrite media, to study Raman and fluorescent scattering by active molecules embedded in a uniaxial bianisotropic-ferrite medium, and to understand the physical process of this class of media. Applications of the present formulations in analysing the electromagnetic scatter, propagation, resonance, and radiation phenomena relevant to the uniaxial bianisotropic-ferrite media are under investigation, and will be reported in the near future.

Acknowledgments

One of the authors (DC) appreciates the help and encouragement of Professor Wiegan Lin and Professor Yushen Zhao. The authors greatly thank two anonymous referees for providing constructive comments, which have improved the quality of this manuscript. This work was partially supported by National Natural Science Foundation of China, and Shanghai R& D Foundation for Applied Material.

Appendix A. Explicit expressions for the spectral parameters of equation (9)

Straightforward mathematical manipulation to equation (6) results in the qth eigenwaves (9), where the explicit expressions for the spectral parameters are found to be

$$A_q^\sigma(k_\rho, k_z) = k_\rho[(k_z - f)(\varepsilon' k_\rho^2 + k_z^2 - a) + e(bk_z + c)]/E_q^\sigma(k_\rho, k_z) \tag{A1}$$

$$B_q^\sigma(k_\rho, k_z) = k_\rho[e(k_z^2 - a) - (bk_z + c)(k_z - f)]/E_q^\sigma(k_\rho, k_z) \tag{A2}$$

$$E_q^\sigma(k_\rho, k_z) = (k_z^2 - a)(\varepsilon' k_\rho^2 + k_z^2 - a) + (bk_z + c)^2 \tag{A3}$$

$$C_q^\sigma(k_\rho, k_z) = \frac{1}{\omega\varepsilon_t}[\mathrm{i}k_0\alpha A_q^\sigma(k_\rho, k_z) + (\mathrm{i}k_0\beta + k_z)B_q^\sigma(k_\rho, k_z)] \tag{A4}$$

and

$$D_q^\sigma(k_\rho, k_z) = \frac{1}{\omega\varepsilon_t}[\mathrm{i}k_0\alpha B_q^\sigma(k_\rho, k_z) + k_\rho - (\mathrm{i}k_0\beta + k_z)A_q^\sigma(k_\rho, k_z)]. \tag{A5}$$

Appendix B. Definition of the cylindrical vector wavefunctions

The cylindrical vector wavefunctions used here are defined as

$$\boldsymbol{M}_n^{(j)}(k_\rho, k_z) = \nabla \times (\boldsymbol{\Psi}_n^{(j)}(k_\rho, k_z)\boldsymbol{e}_z] \tag{B1}$$

$$\boldsymbol{N}_n^{(j)}(k_\rho, k_z) = \frac{1}{k_q}\nabla \times \boldsymbol{M}_n^{(j)}(k_\rho, k_z) \tag{B2}$$

$$\boldsymbol{L}_n^{(j)}(k_\rho, k_z) = \nabla\boldsymbol{\Psi}_n^{(j)}(k_\rho, k_z) \tag{B3}$$

where the generating function is

$$\boldsymbol{\Psi}_n^{(j)}(k_\rho, k_z) = Z_n^{(j)}(k_\rho, \rho)\mathrm{e}^{-\mathrm{i}(k_z z_+ n\phi)} \tag{B4}$$

and

$$Z_n^{(j)}(k_\rho\rho) = \begin{cases} J_n(k_\rho\rho) & j = 1 \\ Y_n(k_\rho\rho) & j = 2 \\ H_n^{(1)}(k_\rho\rho) & j = 3 \\ H_n^{(2)}(k_\rho\rho) & j = 4. \end{cases} \tag{B5}$$

References

[1] Hansen W 1935 *Phys. Rev.* **47** 139
[2] Felsen L B and Marcuvitz N 1973 *Radiation and Scattering of Waves* (Englewood Cliffs, NJ: Prentice-Hall)
[3] Morse P and Feshbach H 1953 *Methods of Theoretical Physics* (New York: McGraw-Hill)
[4] Tai C T 1971 *Dyadic Green's Functions in Electromagnetic Theory* (New York: Intertext)
[5] Fonseca S B A and Giarola A J 1983 *IEEE* **AP-31** 248
[6] Araki K and Itoh T 1981 *IEEE* **AP-29** 84
[7] Bhattacharyya A K and Gary R 1985 *IEEE* **AP-33** 369
[8] Kisliuk M 1980 *IEEE* **MTT-28** 849
[9] Cheng D 1995 *J. Phys. D: Appl. Phys.* **28** 246
[10] Cheng D 1994 *Phys. Rev.* E **50** 4107
[11] Cheng D 1995 *PhD Dissertation* University of Electronic Science and Technology of China
[12] Cheng D, Lin W and Zhao Y 1995 *J. Appl. Phys.* **77** 2889
[13] Tai C T 1965 *IEEE* **AP-13** 322
[14] Tai C T 1991 *Generalized Vector and Dyadic Analysis* (New Jersey: IEEE)
[15] Engheta N and Kowarz M W 1990 *J. Appl. Phys.* **67** 639
[16] Newman E H 1988 *Proc. IEEE* **76** 270
[17] Michalski K A, Nevels R D and Zheng D 1990 *IEEE* **AP-38** 732
[18] Chow H, McNulty P J and Kerker M 1976 *Phys. Rev.* A **13** 396
[19] Videen C, Bickel W S and Boyer J M 1991 *Phys. Rev.* A **43** 5655
[20] Chew W C 1990 *Waves and Fields in Inhomogeneous Media* (New York: Van Nostrand)
[21] Monzon J C 1993 *IEEE* **MTT-41** 1995
[22] Engheta N (ed) 1992 *J. Electromagn. Waves. Applic.* **6** 537
[23] Lindell I V, Sihvola A H, Tretyakov S A and Viitanen A J 1994 *Electromagnetic Waves in Chiral and Bi-Isotropic Media* (Norwood, MA: Artech House)
[24] Ren W 1994 *J. Appl. Phys.* **75** 30
[25] Yin W 1993 *Chinese J. Radio Sci.* **8** 29 (in Chinese)
[26] Kong J A 1975 *Theory of Electromagnetic Waves* (New York: Wiley)
[27] Weiglhofer W S 1994 *J. Phys. A: Math. Gen.* **27** L871
[28] Lindell I V, Viitanen A J and Koivisto P K 1993 *Microw. Opt. Technol. Lett.* **6** 478
[29] Tretyakov S A and Sochava A 1993 *Electron. Lett.* **29** 1048
[30] Lindell I V and Viitanen A J 1993 *Electron. Lett.* **29** 152
[31] Lindell I V, Tretyakov S A and Viitanen A J 1993 *Microw. Opt. Technol. Lett.* **6** 517
[32] Post E J 1962 *Formal Structures of Electromagnetics* (Amsterdam: North-Holland)
[33] Chen H C 1983 *Theory of Electromagnetic Waves—A Coordinate Free Approach* (New York: McGraw-Hill)
[34] Lindell I V and Weiglhofer W S 1994 *IEEE* **AP-42** 1013
[35] Weiglhofer W S and Lindell I V 1994 *Arch. Elektr. Uebertr.* **48** 116
[36] Weiglhofer W S 1994 *Int. J. Electron.* **77** 105
[37] Olyslager F 1995 *IEEE* **AP-43** 430
[38] Lax M and Nelson N F 1973 *Phys. Rev.* B **4** 3694
[39] Kong J A 1981 *Research Topics in Electromagnetic Wave Theory* (New York: Wiley)
[40] Pogorzelski R J and Lun E 1976 *Radio. Sci.* **11** 753
[41] Ren W 1993 *Phys. Rev.* E **47** 4439
[42] Ren W 1993 *J. Math. Phys.* **34** 5376
[43] Ren W 1994 *J. Acoust. Soc. Am.* **95** 1741